

Closing Tues: HW 10.1

Closing Thurs: HW 10.2

Exam 1 will be returned Tues

10.1 Curve Sketching (relative max/min)

Entry Task: Consider the given $y = f(x)$ graph.

What can you conclude about $f'(x)$ at each of the given points?

A $f'(-0.5)$ is positive

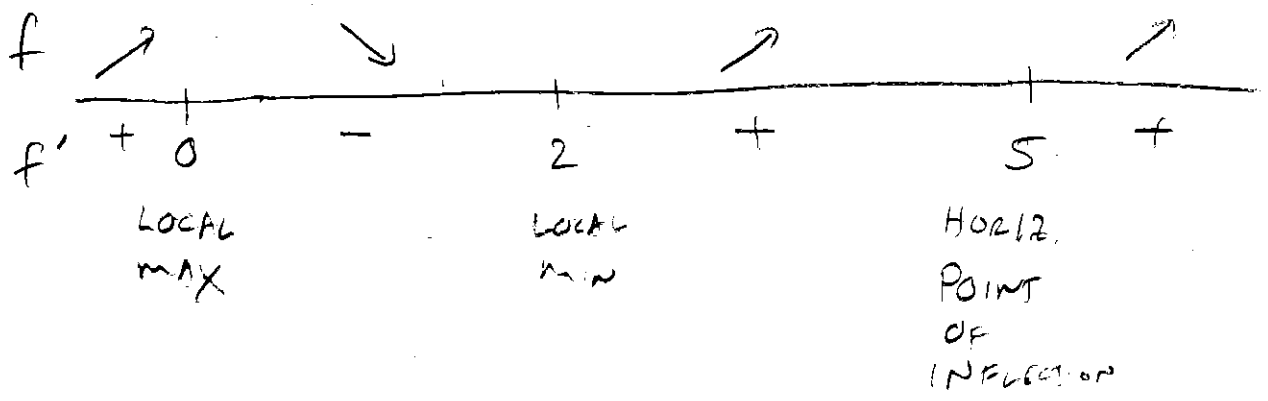
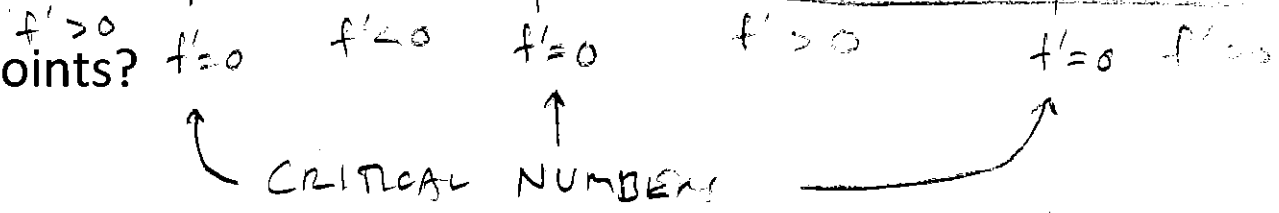
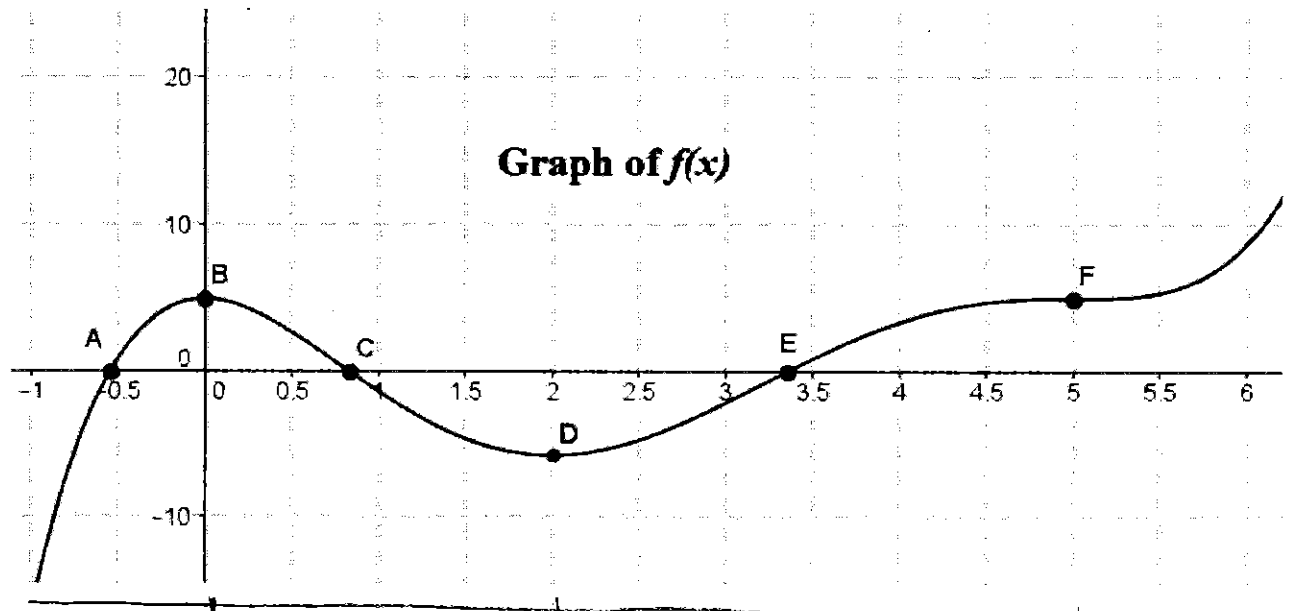
B $f'(0) = 0$

C $f'(0.8)$ is negative

D $f'(2) = 0$

E $f'(3.5)$ is positive

F $f'(5) = 0$



Terminology:

If $f'(x) = 0$ at $x = a$, then we say $x = a$ is a **critical value** (or **critical number**) and

$(x, y) = (a, f(a))$ is call a **critical point**.

We say $y = f(x)$ has a **relative maximum** (or **local maximum**) at $x = a$ if the function changes from increasing to decreasing at $x = a$.

We say $y = f(x)$ has a **relative minimum** (or **local minimum**) at $x = a$ if the function changes from decreasing to increasing at $x = a$.

An **optimum** is a max or min.

We say $y = f(x)$ has a **horizontal point of inflection** at $x = a$ if the function does not change direction (stays inc. to inc. or dec. to dec.).

Examples (from Entry Task)

CRITICAL NUMBERS: $x = 0, x = 2, x = 5$

CRITICAL POINTS:

$$f(0) = 4 \Rightarrow (x, y) = (0, 4)$$

$$f(2) = -5 \Rightarrow (x, y) = (2, -5)$$

$$f(5) = 4 \Rightarrow (x, y) = (5, 4)$$

LOCAL MAXIMUM

[occurs at $x = 0$
local max. value = 4

LOCAL MINIMUM

[occurs at $x = 2$
local min. value = -5

} OPTIMA

HORIZ. POINT OF INFLECTION

[occurs at $x = 5$
h.p.o.i. = $(x, y) = (5, 4)$

Once again don't forget the following table which is fundamental to all applications:

$f(x)$	$f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

NOTES ON SOLVING:

① CLEAR DENOMINATORS

$$\bullet \frac{5}{x} = 2 \Rightarrow 5 = 2x$$

$$\bullet \frac{4}{x^2} - \frac{3}{x} = 1 \Rightarrow 4 - 3x = x^2$$

$$\bullet \frac{3}{5}x - \frac{1}{10} = 0 \Rightarrow 6x - 1 = 0$$

② FACTOR IF POSSIBLE

$$\bullet x^3 - 11x^2 = 0 \Rightarrow x^2(x - 11) = 0$$

$$\bullet x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 4$$

③ REWRITE EXPONENTS & SIMPLIFY

$$\bullet (2x - 1)^{-10}(x - 4) = 0$$

$$\Rightarrow \frac{x - 4}{(2x - 1)^{10}} = 0$$

$$\Rightarrow x - 4 = 0$$

$$\bullet (3x + 4)^{-\frac{1}{2}}(2x) - 10(3x + 4)^{-\frac{1}{2}} = 0$$

$$\Rightarrow (3x + 4)^{-\frac{1}{2}}(2x - 10) = 0$$

$$\Rightarrow \frac{2x - 10}{\sqrt{3x + 4}} = 0$$

$$\Rightarrow 2x - 10 = 0$$

$$\Rightarrow x = 5$$

Example:

Let $f(x) = x^3 - 6x^2 - 15x + 17$

- (a) Find the critical values.
- (b) Find the critical points.

$$f'(x) = 3x^2 - 12x - 15 \stackrel{?}{=} 0$$
$$x^2 - 4x - 5 \stackrel{?}{=} 0 \quad \div 3$$
$$(x - 5)(x + 1) = 0$$

or, QUADRATIC FORMULA, $x = \frac{4 \pm \sqrt{16 - 4(-5)}}{2}$
 $= -1$ or 5

$$x = -1 \Rightarrow f(-1) = (-1)^3 - 6(-1)^2 - 15(-1) + 17$$
$$= -1 - 6 + 15 + 17 = 25$$

$$x = 5 \Rightarrow f(5) = (5)^3 - 6(5)^2 - 15(5) + 17$$
$$= -83$$

(a) CRITICAL VALUES: $x = -1, x = 5$

(b) CRITICAL POINTS = $(-1, 25), (5, -83)$

- (c) Find the intervals over which $f(x)$ is increasing (and over which it is decreasing).
- (d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.

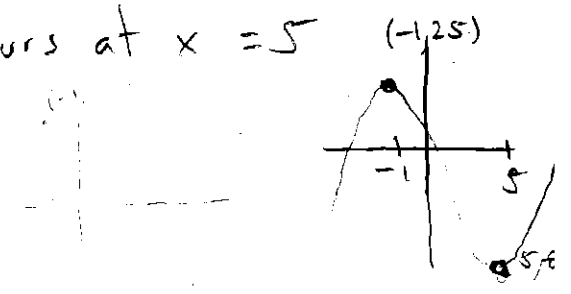
(c)

For $x < -1$: Try $x = -2$
 $f'(-2) = 3(-2)^2 - 12(-2) - 15$
 $= 12 + 24 - 15 > 0$

For $-1 < x < 5$: Try $x = 0$
 $f'(0) = -15 < 0$

For $x > 5$: Try $x = 10$
 $f'(10) = 3(10)^2 - 12(10) - 15$
 $= 300 - 120 - 15 > 0$

(d) CONCLUSIONS:
A local max occurs at $x = -1$.
A local min occurs at $x = 5$



Example:

$$\text{Let } f(x) = 1 + x^3 - \frac{1}{4}x^4$$

- (a) Find the critical values.
(b) Find the critical points.

$$f'(x) = 3x^2 - x^3 \stackrel{?}{=} 0$$

$$x^2(3-x) \stackrel{?}{=} 0$$

$$x^2 = 0 \quad \text{or} \quad 3-x = 0$$
$$x = 0 \quad \quad \quad x = 3$$

(a) CRITICAL NUMBERS: $x=0, x=3$

(b) CRITICAL POINTS:

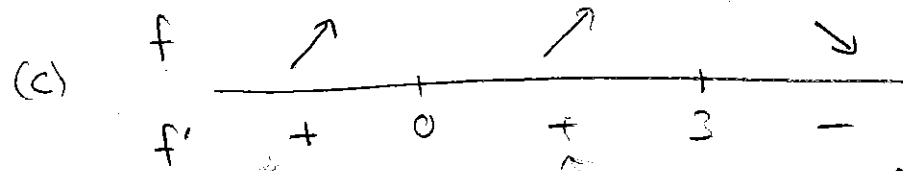
$$x=0 \Rightarrow f(0) = 1 + (0)^3 - \frac{1}{4}(0)^4 = 1$$
$$(0, 1)$$

$$x=3 \Rightarrow f(3) = 1 + (3)^3 - \frac{1}{4}(3)^4 = 28 - \frac{81}{4}$$
$$= 7.75$$

$$(3, 7.75)$$

(c) Find the intervals over which $f(x)$ is increasing (and over which it is decreasing).

(d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.



For $x < 0$: Try $x = -1$

$$f'(-1) = (-1)^2(3 - (-1)) > 0$$

For $0 < x < 3$: Try $x = 1$

$$f'(1) = (1)^2(3 - 1) > 0$$

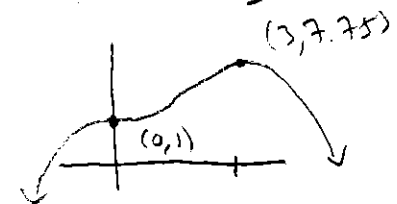
For $x > 3$: Try $x = 10$

$$f'(10) = (10)^2(3 - 10) < 0$$

(d) CONCLUSIONS

$$\text{H.P.O.I.} = (0, 1)$$

LOCAL MAX OCCURS AT $x = 3$



Example:

$$\text{Let } f(x) = \sqrt{x^2 + 6x + 100}$$

- (a) Find the critical values.
- (b) Find the critical points.

$$f'(x) = \frac{1}{2} (x^2 + 6x + 100)^{-\frac{1}{2}} \cdot (2x + 6)$$
$$= \frac{2x + 6}{2\sqrt{x^2 + 6x + 100}} = \frac{x + 3}{\sqrt{x^2 + 6x + 100}} \stackrel{?}{=} 0$$

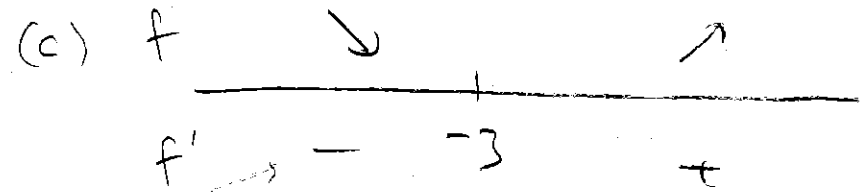
$$\Rightarrow x = -3$$

(a) Critical values: $x = -3$

$$(b) f(-3) = \sqrt{9 - 18 + 100} = \sqrt{91}$$
$$\approx 9.53939$$

Critical points: $(-3, \sqrt{91})$

- (c) Find the intervals over which $f(x)$ is increasing (and over which it is decreasing).
- (d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.



For $x < -3$: Try $x = -10$

$$f'(-10) = \frac{-10 + 3}{\sqrt{(-10)^2 + 6(-10) + 100}} < 0$$

For $x > -3$: Try $x = 0$

$$f'(0) = \frac{0 + 3}{\sqrt{100}} > 0$$

(d) LOCAL MIN occurs at $x = -3$

